

## AMENDMENTS TO THE CLAIMS

1-18. (Cancelled)

19. (New) A method for modelling, analyzing and/or synthesizing, a windowed signal, comprising computing simultaneously the frequencies and complex amplitudes from the signal using a nonlinear least squares method, whereby the computational complexity is reduced by taking into account the bandlimited property of the window resulting in band-diagonal system matrices for the computation of the amplitudes and frequency optimization step.

20. (New) The method according to claim 19 using a stationary nonharmonic signal model according to (Eq. (2)):

$$\tilde{x}_n = \Re \left[ w_n \sum_{k=0}^{K-1} A_k \exp(-2\pi i \omega_k \frac{n-n_0}{N}) \right] \quad (2)$$

which is a model with  $K$  stationary components where each component is characterized by its complex amplitude  $A_k$  and frequency  $\omega_k$  where  $w_n$  is the window of claim 1; or

an harmonic signal model according to (Eq. (3)):

$$\tilde{x}_n = \Re \left[ w_n \sum_{k=0}^{S-1} \sum_{p=0}^{S_k-1} A_{k,p} \exp(-2\pi i p \omega_k \frac{n-n_0}{N}) \right] \quad (3)$$

which is a model with  $S$  quasi-periodic stationary sound sources with a fundamental frequency  $\omega_k$ , each consisting of  $S_k$  sinusoidal components with frequencies that are integer multiples of  $\omega_k$ , in which the complex amplitude of the  $p$ th component of the  $k$ th source is denoted  $A_{k,p}$ , and where  $w_n$  is the window of claim 1.

21. (New) The method according to claim 19 using a nonstationary nonharmonic model according to (Eq. (4)):

$$\tilde{x}_n = \Re \left[ w_n \sum_{k=0}^{K-1} \sum_{p=0}^{P-1} A_{k,p} (-2\pi i \frac{n-n_0}{N})^p \exp(-2\pi i \omega_k \frac{n-n_0}{N}) \right] \quad (4)$$

which is a model with  $K$  nonstationary sinusoidal components which have independent frequencies  $\omega_k$ , in which the amplitudes  $A_{k,p}$  denote the  $p$ -th order of the  $k$ -th sinusoid, and where  $w_n$  is the window of claim 1.

22. (New) The method according to claim 20, comprising the computation of the spectrum as a linear combination of the frequency responses of the window according to (Eq. (11)):

$$\tilde{X}_m = \sum_{k=0}^{K-1} A_k W(m + \omega_k) \quad (11)$$

for the stationary nonharmonic model,  
or (Eq. (12)):

$$\tilde{X}_m = \sum_{k=0}^{S-1} \sum_{p=0}^{S_k-1} A_{k,p} W(m + p\omega_k) \quad (12)$$

for the harmonic model,

where the fourier transform of a complex signal results in a spectrum  $\tilde{X}_m$ , where  $W(m)$  denotes the discrete time fourier transform of  $w_n$  and whereby only the main lobes of the responses are computed by using look-up tables.

23. (New) The method according to claim 21, comprising the computation of the spectrum as a linear combination of the frequency responses of the window according to (Eq. (13)):

$$\begin{aligned} \tilde{X}_m &= \sum_{n=0}^{N-1} w_n \left[ \sum_{k=0}^{K-1} \sum_{p=0}^{P-1} A_{k,p} (-2\pi i \frac{n-n_0}{N})^p \exp(-2\pi i \omega_k \frac{n-n_0}{N}) \right] \exp(-2\pi i m \frac{n-n_0}{N}) \\ &= \sum_{k=0}^{K-1} \sum_{p=0}^{P-1} A_{k,p} \left[ \sum_{n=0}^{N-1} w_n (-2\pi i \frac{n-n_0}{N})^p \exp(-2\pi i (\omega_k + m) \frac{n-n_0}{N}) \right] \\ &= \sum_{k=0}^{K-1} \sum_{p=0}^{P-1} A_{k,p} \frac{\partial^p}{\partial m^p} W(\omega_k + m) \end{aligned} \quad (13)$$

for the nonstationary model, where the fourier transform of a complex signal results in a spectrum  $\tilde{X}_m$ , where  $W(m)$  denotes the discrete time fourier transform of  $w_n$  whereby only the main lobes of the responses are computed by using look-up tables.

24. (New) The method according to claim 20, comprising the step of computing the stationary complex amplitudes, by solving the equations (Eq. (19)):

$$\begin{bmatrix} B^{1,1} & B^{1,2} \\ B^{2,1} & B^{2,2} \end{bmatrix} \begin{bmatrix} A^r \\ A^i \end{bmatrix} = \begin{bmatrix} C^1 \\ C^2 \end{bmatrix} \quad (19)$$

where

$$\begin{aligned} B_{l,k}^{1,1} &= \sum_{n=0}^{N-1} w_n^2 \cos(2\pi\omega_k \frac{n-n_0}{N}) \cos(2\pi\omega_l \frac{n-n_0}{N}) \\ B_{l,k}^{1,2} &= \sum_{n=0}^{N-1} w_n^2 \sin(2\pi\omega_k \frac{n-n_0}{N}) \cos(2\pi\omega_l \frac{n-n_0}{N}) \\ B_{l,k}^{2,1} &= \sum_{n=0}^{N-1} w_n^2 \cos(2\pi\omega_k \frac{n-n_0}{N}) \sin(2\pi\omega_l \frac{n-n_0}{N}) \\ B_{l,k}^{2,2} &= \sum_{n=0}^{N-1} w_n^2 \sin(2\pi\omega_k \frac{n-n_0}{N}) \sin(2\pi\omega_l \frac{n-n_0}{N}) \\ C_l^1 &= \sum_{n=0}^{N-1} x_n w_n \cos(2\pi\omega_l \frac{n-n_0}{N}) \\ C_l^2 &= \sum_{n=0}^{N-1} x_n w_n \sin(2\pi\omega_l \frac{n-n_0}{N}) \end{aligned}$$

using (Eq. (20)):

$$\begin{aligned} B_{l,k}^{1,1} &= \frac{1}{2} \Re(Y(\omega_k + \omega_l)) + \frac{1}{2} \Re(Y(\omega_k - \omega_l)) \\ B_{l,k}^{1,2} &= -\frac{1}{2} \Im(Y(\omega_k + \omega_l)) - \frac{1}{2} \Im(Y(\omega_k - \omega_l)) \\ B_{l,k}^{2,1} &= -\frac{1}{2} \Im(Y(\omega_k + \omega_l)) + \frac{1}{2} \Im(Y(\omega_k - \omega_l)) \\ B_{l,k}^{2,2} &= -\frac{1}{2} \Re(Y(\omega_k + \omega_l)) + \frac{1}{2} \Re(Y(\omega_k - \omega_l)) \\ C_l^1 &= \Re\left(\frac{1}{N} \sum_{m=0}^{N-1} X_m W(m + \omega_l)\right) \\ C_l^2 &= -\Im\left(\frac{1}{N} \sum_{m=0}^{N-1} X_m W(m + \omega_l)\right) \end{aligned} \quad (20)$$

such that only the elements around the diagonal of  $\mathbf{B}$  are taken into account, whereby a shifted form  $\overleftarrow{\mathbf{B}}$  is computed containing only  $D$  diagonal bands of  $\mathbf{B}$  according to (Eq. (27)):

$$\begin{aligned}\overleftarrow{B}^{1,1}_{l,k} &= B^{1,1}_{l,l+k-D} \\ \overleftarrow{B}^{2,2}_{l,k} &= B^{2,2}_{l,l+k-D}\end{aligned}\quad (27)$$

and Eq. (20), whereby the computation of the Eq. (20) requires the computation of the frequency response of the window and the square window denoted by  $W(m)$  and  $Y(m)$  respectively, and solving equation given by Eq. (19) directly from  $\overleftarrow{B}$  and  $C$  in (Eq. (28)):

$$\begin{aligned}A^r &= \text{SOLVE}(\overleftarrow{B}^{1,1}, C^1) \\ A^i &= \text{SOLVE}(\overleftarrow{B}^{2,2}, C^2)\end{aligned}\quad (28)$$

by an adapted gaussian elimination procedure.

25. (New) The method according to claim 20, further comprising the step of optimizing the frequencies for the stationary nonharmonic model by solving the equation (Eq. (34)):

$$H\Delta\omega = h \quad (34)$$

using (Eq. (42)):

$$\begin{aligned}\Delta\omega_l &= \hat{\omega}_l - \omega_l \\ h_l &= -\frac{2}{N} \Re \left( A_l \sum_{m=0}^{N-1} R_m W'(\hat{\omega}_l - m) \right) \\ H_{lk} &= \Re(A_k A_l Y''(\hat{\omega}_k + \hat{\omega}_l)) - \Re(A_k A_l^* Y''(\hat{\omega}_k - \hat{\omega}_l)) \\ &\quad - \lambda_1 \delta_{kl} \frac{2}{N} \Re \left( A_l \sum_{m=0}^{N-1} R_m W''(\hat{\omega}_l - m) \right) + \delta_{kl} \lambda_2\end{aligned}\quad (42)$$

such that only elements around the diagonal of  $H$  are taken into account, whereby a shifted form  $\overleftarrow{H}$  is computed containing only  $D$  diagonal bands according to (Eq. (36)):

$$\overleftarrow{H}_{lk} = H_{l,l+k-D} \quad (36)$$

and Eq. (42), whereby the gradient  $h$  is computed from the residual spectrum  $R_m$ , where  $R_m = X_m - \tilde{X}_m$  denotes the spectrum of the residual  $r_m$ , and from amplitude  $A_l$  and frequencies  $\omega_l$ , and requires the computation of derivative of the frequency response of the window  $W'(m)$ , whereby the first term of  $H$  requires the computation of the second derivative of the frequency response of the square window denoted  $Y''(m)$ , whereby the second term of  $H$  is computed from the residual spectrum  $R_m$ , amplitude  $A_l$  and frequencies  $\omega_l$ , and requires the computation of the second derivative of the frequency

response  $W''(m)$ , whereby the parameter  $\lambda_1$  allows to switch between different optimization methods and the parameter  $\lambda_2$  regularizes the system matrix, and computing the optimization step by solving the system of equations directly on  $\bar{\mathbf{H}}$  and  $\mathbf{h}$  according to (Eq. (37)):

$$\Delta\bar{\omega} = \text{SOLVE}(\bar{\mathbf{H}}, \mathbf{h}) \quad (37)$$

by an adapted gaussian elimination procedure.

26. (New) The method according to claim 20, further comprising the step of optimizing the frequencies for the harmonic signal model, by computing the optimization step solving (Eq. (48)):

$$\mathbf{H}\Delta\omega = \mathbf{h}$$

using Eq  
with

$$\begin{aligned} \Delta\omega_l &= \hat{\omega}_l - \omega_l \\ h_l &= -\frac{2}{N} \sum_{q=1}^{S_l-1} \Re \left( \sum_{m=0}^{N-1} R_m q A_{l,q} W'(q\omega_l - m) \right) \\ H_{l,k} &= \sum_{q=1}^{S-1} \left[ \sum_{r=1}^{r_{max,1}} q r \Re(A_{p,q} A_{l,r} Y''(q\omega_p + r\omega_l)) \right. \\ &\quad + \sum_{r=r_{min,2}}^{r_{max,2}} q r \Re(A_{p,q} A_{l,r} Y''(q\omega_p + r\omega_l)) \\ &\quad \left. - \sum_{r=r_{min,3}}^{r_{max,3}} q r \Re(A_{p,q} A_{l,r}^* Y''(q\omega_p - r\omega_l)) \right] \\ &\quad - \lambda_1 \delta_{lp} \frac{2}{N} \Re \left( \sum_{q=1}^{S_l-1} \sum_{m=0}^{N-1} R_m q^2 A_{p,q} W''(q\omega_p - m) \right) \end{aligned}$$

whereby the gradient  $\mathbf{h}$  is computed from the residual spectrum  $R_m$ , where  $R_m = X_m - \tilde{X}_m$  denotes the spectrum of the residual  $r_n$  and  $W'(m)$ , and from amplitude  $A_l$  and frequencies  $\omega_l$ , and requires the computation of derivative of the frequency response of the window  $W'(m)$ , whereby the first term of  $\mathbf{H}$  requires the computation of the second derivative of the frequency response of the square window denoted  $Y''(m)$ , whereby the second term of  $\mathbf{H}$  is computed from the residual spectrum  $R_m$ , amplitude  $A_l$

and frequencies  $\omega_l$ , and requires the computation of the second derivative of the frequency response  $W''(m)$ , whereby the parameter  $\lambda_1$  allows to switch between different optimization methods and the parameter  $\lambda_2$  regularizes the system matrix.

27. (New) The method according to claim 19, further comprising the step of computing the polynomial complex amplitudes by solving the equation (Eq. (55)):

$$\begin{bmatrix} \mathbf{B}^{1,1} & \mathbf{B}^{1,2} \\ \mathbf{B}^{2,1} & \mathbf{B}^{2,2} \end{bmatrix} \begin{bmatrix} \mathbf{A}^1 \\ \mathbf{A}^2 \end{bmatrix} = \begin{bmatrix} \mathbf{C}^1 \\ \mathbf{C}^2 \end{bmatrix} \quad (55)$$

using (Eq. (63)):

$$\begin{aligned} B_{lP+q,kP+p}^{1,1} &= \frac{1}{2} \left[ \frac{\partial^{p+q}}{\partial m^{p+q}} \Re[Y(m)] \right]_{m=\omega_k+\omega_l} + (-1)^{-q} \frac{1}{2} \left[ \frac{\partial^{p+q}}{\partial m^{p+q}} \Re[Y(m)] \right]_{m=\omega_k-\omega_l} \\ B_{lP+q,kP+p}^{1,2} &= -\frac{1}{2} \left[ \frac{\partial^{p+q}}{\partial m^{p+q}} \Im[Y(m)] \right]_{m=\omega_k+\omega_l} - (-1)^q \frac{1}{2} \left[ \frac{\partial^{p+q}}{\partial m^{p+q}} \Im[Y(m)] \right]_{m=\omega_k-\omega_l} \\ B_{lP+q,kP+p}^{2,1} &= -\frac{1}{2} \left[ \frac{\partial^{p+q}}{\partial m^{p+q}} \Im[Y(m)] \right]_{m=\omega_k+\omega_l} + (-1)^q \frac{1}{2} \left[ \frac{\partial^{p+q}}{\partial m^{p+q}} \Im[Y(m)] \right]_{m=\omega_k-\omega_l} \\ B_{lP+q,kP+p}^{2,2} &= -\frac{1}{2} \left[ \frac{\partial^{p+q}}{\partial m^{p+q}} \Re[Y(m)] \right]_{m=\omega_k+\omega_l} + (-1)^{-q} \frac{1}{2} \left[ \frac{\partial^{p+q}}{\partial m^{p+q}} \Re[Y(m)] \right]_{m=\omega_k-\omega_l} \\ C_{lP+q}^1 &= \Re \left( \frac{1}{N} \sum_{m=0}^{N-1} X_m \frac{\partial^q}{\partial m^q} W(m + \omega_l) \right) \\ C_{lP+q}^2 &= -\Im \left( \frac{1}{N} \sum_{m=0}^{N-1} X_m \frac{\partial^q}{\partial m^q} W(m + \omega_l) \right) \\ A_{kP+p}^1 &= A_{k,p}^r \\ A_{kP+p}^2 &= A_{k,p}^i \end{aligned} \quad (63)$$

such that only the elements around the diagonal of  $\mathbf{B}$  are taken into account, whereby a shifted form  $\overleftarrow{\mathbf{B}}$  is computed containing only  $PD$  diagonal bands of  $\mathbf{B}$  according to (Eq. (64)):

$$\begin{aligned} \overleftarrow{B}^{1,1}_{lP+q,kP+p} &= B_{lP+q,lP+q+kP+p-(D+1)P+1}^{1,1} = B_{lP+q,(k+l-D)P+(p+q-P+1)}^{1,1} \\ \overleftarrow{B}^{2,2}_{lP+q,kP+p} &= B_{lP+q,lP+q+kP+p-(D+1)P+1}^{2,2} = B_{lP+q,(k+l-D)P+(p+q-P+1)}^{2,2} \end{aligned} \quad (64)$$

and Eq. (63), whereby the computation is required of the frequency response of the square window and its derivatives  $\frac{\partial^p}{\partial m^p} Y(m)$  whereby the computation is required of the frequency response of the window and its derivatives  $\frac{\partial^p}{\partial m^p} W(m)$ , and solving the equation given by Eq. (55) directly from  $\overleftarrow{\mathbf{B}}$  and  $\mathbf{C}$  by an adapted gaussian elimination procedure.

28. (New) The method according to claim 24 or 27, comprising a preprocessing step which comprises:

sorting the frequencies to obtain a band diagonal matrix  $D$ , determining the number of diagonal bands  $D$  being defined as the largest  $k - l$  for which  $-\beta \leq \omega_k - \omega_l \leq \beta$ , where  $\omega_k$  and  $\omega_l$  denote two frequency values and  $\beta$  the width of the main lobe of the frequency response of the window.

29. (New) The method according to claim 19, further comprising the step of computing instantaneous frequencies and instantaneous amplitudes according to (Eq. (69)):

$$\begin{aligned} \Psi_k(n) &= \sqrt{\left(\sum_{p=0}^{P-1} \hat{A}_{k,p}^r (-2\pi i \frac{n-n_0}{N})^p\right)^2 + \left(\sum_{p=0}^{P-1} \hat{A}_{k,p}^i (-2\pi i \frac{n-n_0}{N})^p\right)^2} \\ \Phi_k(n) &= 2\pi i \omega_k \frac{n-n_0}{N} + i \arctan \left( \frac{\sum_{p=0}^{P-1} \hat{A}_{k,p}^i (-2\pi i \frac{n-n_0}{N})^p}{\sum_{p=0}^{P-1} \hat{A}_{k,p}^r (-2\pi i \frac{n-n_0}{N})^p} \right) \end{aligned} \quad (69)$$

whereby the instantaneous frequency can be used as a frequency estimate for the next iteration as expressed in (Eq. (73)):

$$\omega_k^{(r+1)} = \omega_k^{(r)} - \left(\frac{1}{N}\right) \frac{\hat{A}_{k,0}^r \hat{A}_{k,1}^i - \hat{A}_{k,0}^i \hat{A}_{k,1}^r}{\hat{A}_{k,0}^i{}^2 + \hat{A}_{k,0}^r{}^2} \quad (73)$$

30. (New) The method according to claim 19, further comprising the step of computing damping factor according to (Eq. (78)):

$$\rho_k \approx -\left(\frac{2\pi}{N}\right) \frac{\hat{A}_{k,0}^r \hat{A}_{k,1}^r + \hat{A}_{k,0}^i \hat{A}_{k,1}^i}{\hat{A}_{k,0}^i{}^2 + \hat{A}_{k,0}^r{}^2} \quad (78)$$

in case that the amplitudes are exponentially damped.

31. (New) The method according to claim 19, where a scaled frequency response is used for the analysis of a zero padded window, where  $W^M(m - m_0)$  denotes the frequency response of the window of length  $M$  and  $W_M^N(m - n_0) = W^M(\frac{M}{N}m - m_0)$  the zero padded version of this window up to a length  $N$ , and  $w_{M'}^{N'}(n - n_0) = \frac{N}{N'} W^M(\frac{N}{N'}n - m_0)$  the inverse transform of the truncated spectrum to a length  $N'$  reducing the window length to

$M' = \frac{M}{N} N'$ , resulting in a scaled and zero padded version of the window by computing the inverse transform of the scaled frequency response yielding (Eq. (1)):

$$\frac{N'}{N} w_{M'}^{N'}(n - n'_0) = \frac{1}{N} \sum_{m=0}^{N'-1} W^M\left(\frac{M'}{N'} m - m_0\right) \exp\left(2\pi i \frac{(n - n'_0)(m - m_0)}{N'}\right) \quad (1)$$

32. (New) The method according to claim 19 for accurate pitch estimation, wherein the windowed signal is a sound having a pitch and the method further comprises accurately estimating said pitch based on the computed frequencies and complex amplitudes.

33. (New) The method according to claim 19, wherein the method is applied to determine arbitrary sample rate conversion.

34. (New) The method according to claim 19, wherein the windowed signal is a sound and wherein noise residual, the amplitudes and the frequencies are encoded in a bitstream which is stored, broadcasted or transmitted at a sender side of a parametric/sinusoidal audio coder, and a receiver decodes the bitstream back to the parameters and synthesizes the sound

35. (New) The method according to claim 19 for audio effects whereby noise  $r_n$ , the amplitudes  $\bar{A}$  and the frequencies  $\bar{\omega}$  are manipulated by an effects processor yielding  $r_n^*$ ,  $\bar{A}^*$  and  $\bar{\omega}^*$  and synthesized with these modified parameters.

36. (New) The method according to claim 19 for source separation, whereby sinusoidal components originating from the same sound source are grouped and synthesized separately.

37. (New) The method according to claim 19 for automated annotation and transcription whereby the signal is segmented according to the values of the amplitudes and frequencies.